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New Research in Early Gothic Flying Buttresses

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1. INTRODUCTION

The twelfth-century builders who first explored the architectural style which we now call Gothic did far more than respond to the latest trends in architectural fashion: their intensive experiments engendered a profound transformation of building practice and structural understanding. Gothic architecture, as a result, is rightfully considered one of the most important moments in the history of building construction. Among the architectural innovations made by these builders, the flying buttress played a pivotal role: by efficiently removing thrust, concentrated at specific points on the upper walls of Gothic buildings, to far-removed supports, the flying buttress made it possible to transform, over the course of the late-twelfth and thirteenth centuries, the heavy upper wall and small window of the Romanesque building into a soaring cage of glass.

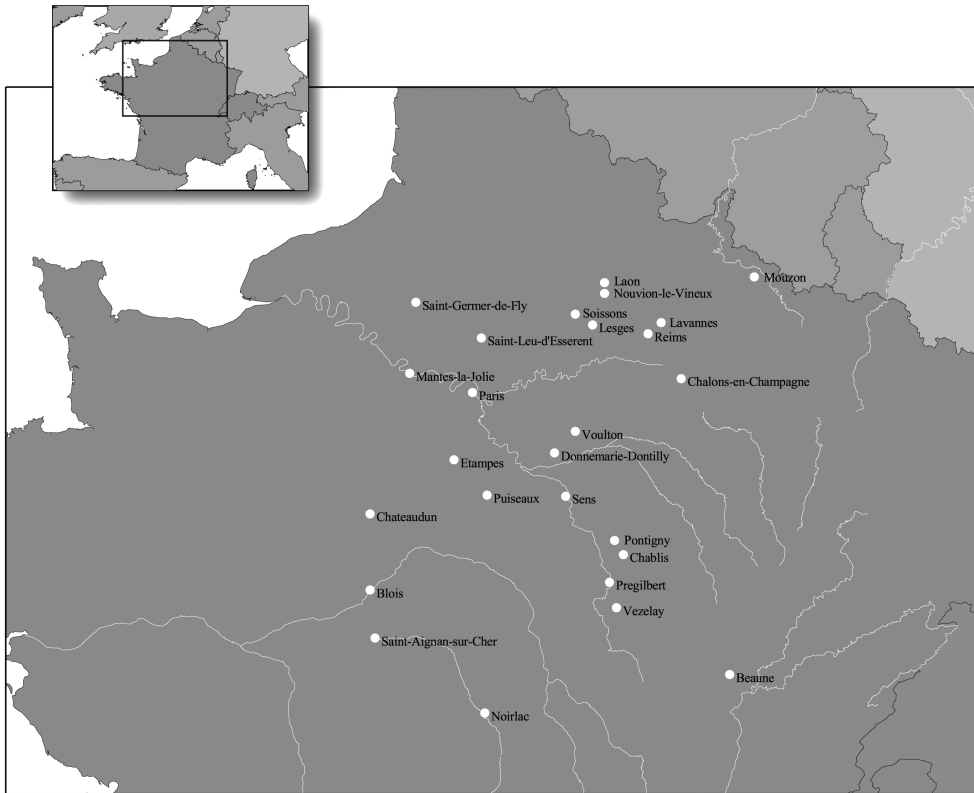


Figure 1. Geographical locations of the buildings studied

The study of this critical structural device has remained essentially in the realm of art history. Apart from a handful of articles which grapple with their art historical problems directly — see, for example, Lefèvre-Pontalis (1919), Prache (1976), Henriet (1982), Plagnieux (1992), or Murray (1998) — flying buttresses are most often mentioned in passing, in the context of a general building description. They are described formally, and then often placed into a sequence of *evolution* — a concept that must be treated with the greatest care, given both the problematic nature of the term and the reality of the maelstrom of simultaneous architectural experimentation that prevailed in twelfth-century France. More recent studies have brought greater nuance (Plagnieux, 2000, for example), and have most importantly provided firmer dates for many of the key buildings. But many questions remain: were these medieval-looking flying buttresses visible on twelfth-century buildings in France and England in fact built at the same time as the building they ostensibly supported? Are formal similarities among flyers a reliable indicator of date or region of construction? Can specific design principles such as structural efficiency be discerned among early Gothic flying buttresses? Archeological evidence, while the only truly reliable index in the absence of documents, is all too often compromised by the vagaries of time and requisite restorations. But one of the most important sources of information has gone largely untapped: the performance of an element which, despite its subsequent aesthetic reception, is first and foremost structural. This paper presents the geometry and structural behavior of 33 flying buttresses whose forms (the actual flying buttresses are most often heavily restored or replaced) can safely be assumed to date from the mid-twelfth century to the early thirteenth century (**figure 1**). Structural information is used to supplement the dearth of traditional evidence to make an initial attempt to discern patterns of formal and technical progress.

2. TECHNIQUES OF STRUCTURAL ANALYSIS OF FLYING BUTTRESSES

The theoretical and technical bases for the current study, as well as a review both of the principles of limit analysis as applied to flying buttresses and of previous such studies, can be found in Nikolinakou et al. (2005). Following is a summary of the key assumptions and concepts described therein:

The flying buttress (**figure 2**) is a strut designed to move compressive force (generated by the passive outward thrust of the vaults and active thrusts of wind loading on the roof) to a set of appropriately resistant pylons (the buttresses) situated away from the body of the building. The twelfth-century flying buttress depicted in figure 2 is a good representation of the flyers of that period in general. With only minor formal variations, a twelfth-century flyer is comprised of an arch capped by a flat stone band (the coping). The flying buttress arch abuts the *culée* (a term borrowed from the French for lack of an appropriate English equivalent), which sits atop a buttress or a wall. In what follows, the term *flying buttress* (or *flyer*) is considered not to include the *culée* (**figure 3**).

Because masonry has far greater strength than the range of compressive stresses typically present in Gothic buildings, it is considered incompressible; the flying buttress is modeled as a series of rigid blocks, whose equilibrium conditions can be examined without reference to the constitutive laws of the material. The structural behavior of a flyer is thus a problem of stability, not strength, and, using the principles of static equilibrium, can be evaluated

geometrically. Flying buttress masonry is further considered incapable of resisting tensile forces and is assigned a characteristic unit weight of 24kN/m^3 . Finally, although flying buttresses are generally believed to have sufficient frictional resistance to sliding, actual stability is evaluated on a case-by-case basis.

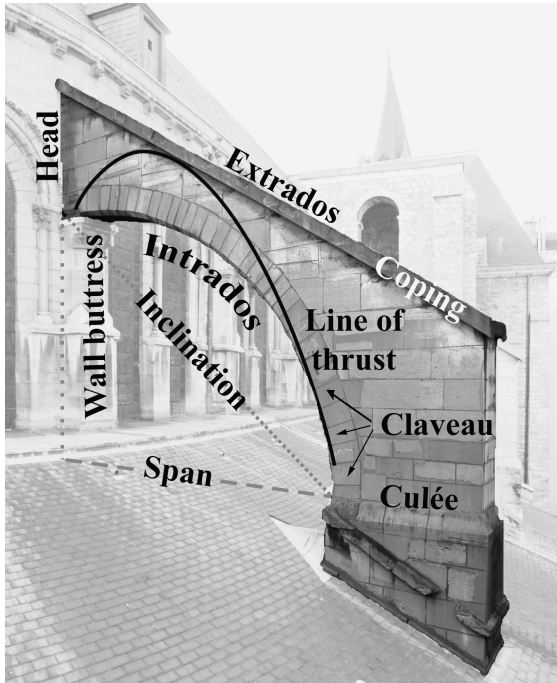


Figure 2. Flying buttress terminology (after Nikolinakou et al, 2005)

Flyers are indeterminate structures, and can exist in equilibrium with an infinite number of possible thrust states, each of which can be represented by a line of compressive force within the confines of the flyer shape (Heyman 1966). Even though the actual present forces are unknown, the limits of structural behavior — the possible minimum and maximum thrusts — can be determined, because they are dictated by the form of the structure.

Looking to the limits is not arbitrary: it was also at the limits of design that the medieval builder received the feedback that taught him how to build. A solid wall makes an excellent buttress because it will transmit thrust effortlessly — but a solid wall is heavy, uneconomical and blocks light. If this wall is voided into the shape of an arch, to make it ‘fly,’ the limits of this new form could come into play, and may even make themselves known — through cracking, displacement, or even collapse — when the tolerances of the design are approached.

For the early Gothic flyers under consideration, the upper limit of force able to be transmitted without collapse depends on the structural performance of neighboring elements, and its calculation would require the study of the complete building cross-section, which is beyond the scope of this study. It is reasonable to assume, based on the form of the flyers studied here, that another element of the cross-sectional structure would fail before any of them did. The study of the upper limit thus does not reveal much about the form or structural performance of the flyer itself.

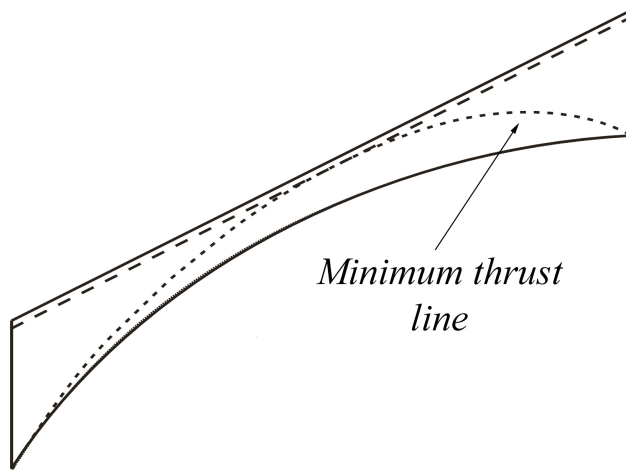


Figure 3. Minimum thrust line

The state of thrust at the lower limit, on the other hand, is established by the flyer form alone, which makes it possible to compare individual flyers rather than entire structural systems. The minimum (or passive) thrust state is the condition in which the flying buttress exerts the smallest possible outward force on its neighboring elements, or, stated in another way, the minimum horizontal force required to keep the claveaux of the flying buttress together. The minimum thrust state is described geometrically by a line of thrust which has the steepest possible rise able to fit within the confines of the flyer shape (**figure 3**). Three criteria of the minimum thrust state in particular are used here to evaluate the effectiveness of the flying buttress:

- a. The lower the minimum thrust, the lesser the load exerted by the flyer onto neighboring structures (clerestory wall, culée), and the larger the outward displacement of its supports able to be sustained before collapse.
- b. A more vertical line of thrust at the culée increases stability, because a horizontally directed force would tend both to overturn the supporting structure and to induce shear stresses in the upper culée masonry.
- c. Conversely, a more horizontal line of thrust at the head increases stability, because a predominantly vertical force component can cause sliding among the first few claveaux, whose joints at the flyer head are nearly vertical.

Structural analyses of the flying buttresses were performed using graphic statics. Though it is true that discrete and finite element analyses offer a number of tools capable of modeling not only the rigid block nature of a flying buttress, but also the slippage and interpenetration between individual stones, graphic static analysis tools remain unsurpassed in their ability to simply and easily represent both the flow of forces through the flyer and concepts of instability or failure (Block, 2005), and are in general more readily comprehensible — a critical consideration for a subject where a strong collaboration between engineers and art historians is required.

3. FORMAL CHARACTERISTICS OF EARLY GOTHIC FLYING BUTTRESSES

A flying buttress can be characterized geometrically in terms of length, inclination, and a representative thickness; the location of the centroid, which represents the distribution of mass, is also considered (see **figure 2** and **table 1**). The length of a flying buttress is dictated by the width of the aisle or gallery over which it must span; its inclination is determined by the point at which it abuts the clerestory wall and the height of the culée. Mean flyer inclination (or flyer angle) is 43 degrees, with an actual range of 26 degrees to 60 degrees. Lengths vary between 2.14 m and 7.81 m, with an average of 4.28 m. The majority of the flyers studied span only a single aisle; others, because of the placement of the culée on the walls or haunches of vaults between outer aisle chapels, span slightly more. Only later in the twelfth century (exactly when is a point of contention) do flyers appear which fully span two aisles (the nave flyers at Notre-Dame of Paris, for example, are roughly 11 m long). Flyer thickness varies from a maximum at the culée to a minimum at the center of the span. The thickness at the centroid ranges from 0.26 m to 2.17 m, while the thickness at the flyer head varies between 0.58 m and 2.75 m. For the thickest flying buttresses, even the minimum thickness can be half the length. Even though flying buttresses are, generally speaking, thicker at the culée, in most cases the centroids of early Gothic flyers lie close to the mid-length of the flyer span, because of the relatively large thickness at the head. Finally, the flying buttress width, a value required for the calculation of mass, which is a principal source of vertical loading in the minimum thrust state, varies from 0.3 m to 1.0 m.

The shape of the intrados is also a useful formal index, because it is related to the centering used for flyer construction. The intrados of the flying buttresses studied can be described by a single circular arc, which extends from the springing of the intrados to the position at which the head meets its point of support. When there is an external clerestory wall passage, as, for example, at Saint-Remi in Reims or Donnemarie (**figure 4**), this point of support is not the clerestory wall, but rather a column or pillar removed from the wall to form a passage. The intrados arc terminates at the flying buttress head support, and continues horizontally to the wall. Thus the flying buttress centering, constructed in a single arc, would have been built in tandem with this head support; the portion of the flyer head which extended beyond, while technically part of the flyer, was constructionally different.



Figure 4. Detail of flying buttress at Donnemarie (photo: Tallon)

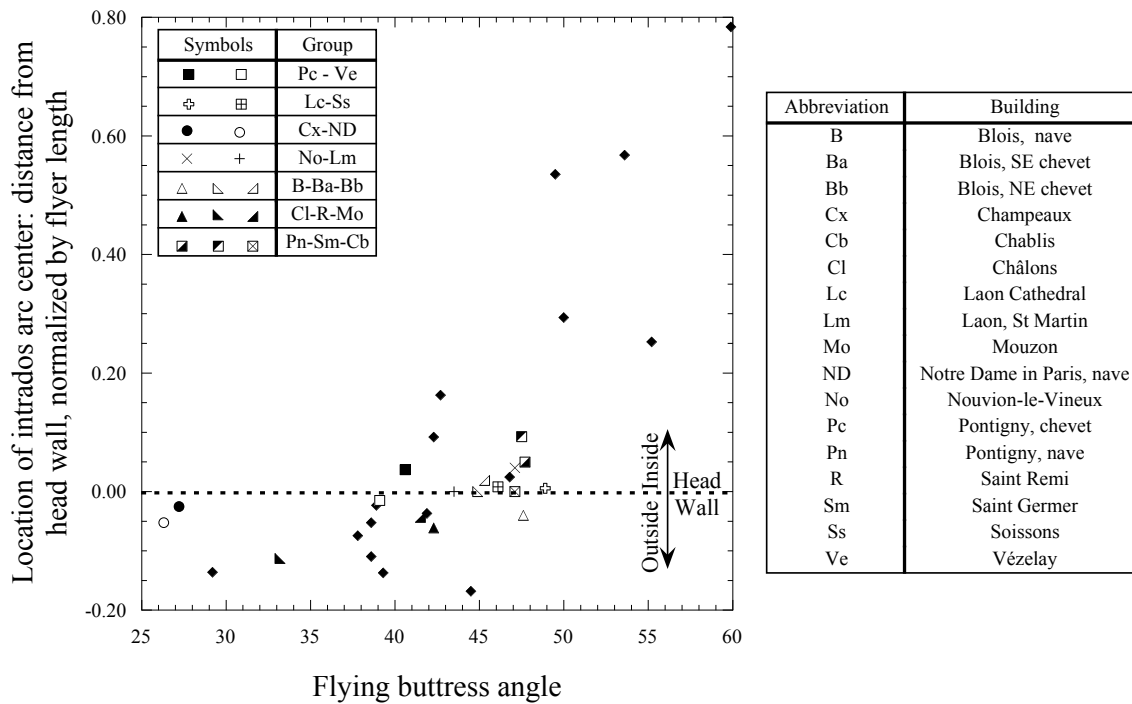


Figure 5. Correlation between flyer inclination and the location of the intrados arc center. Selected flyer groups serve to illustrate the discussion in section 5 below.

Art historians have long maintained that one of the key indices of construction date of Gothic flying buttresses is the form of the intrados of the flyer arch; it is often held that early Gothic flyer intrados segments are described by a quarter-circle. This is not in fact the case. While the intrados of all flyers studied are circular arcs, their central angle varies between

55 and 100 degrees. The location of the center of this arc can also be used as an index. Whereas in the majority of buildings it is located very close to the clerestory wall, in certain cases it moves well inside.

Many of these geometric characteristics are interrelated. **Figure 5**, for example, shows that the flyers with intrados arc centers well inside the clerestory wall are also the steepest; those with shallow angles have centers closer to the culée. Given that steeper flyers tend to have their centroids closer to the springing, a similar relation exists between the locations of the intrados arc center and the centroid — the five flyers with centers well inside the clerestory wall have a higher mass concentration at the culée.

Table 1. Summary of basic geometric information¹

	Minimum	Maximum	Average
Inclination (°)	27.20	59.90	43.70
Length (m)	2.14	7.81	4.31
Thickness at centroid (m)	0.26	2.17	0.92
Thickness at head (m)	0.74	2.68	1.51
Flyer width (m)	0.30	1.00	0.61
Ratio of x_c over length ²	0.37	0.56	0.46
Angle of circular arc	56.50	98.40	83.58
Ratio of x_o over length ³	-0.17	0.78	0.06

¹The flyers at Notre-Dame in Paris are excluded from this table because of their uncertain date

² x_c = the horizontal distance between the centroid and the flyer springing point

³ x_o = the horizontal distance between the head and the center of the arc segment that describes intrados (positive towards the inside of the building)

4. STRUCTURAL BEHAVIOR OF EARLY GOTHIC FLYING BUTTRESSES

As explained above, flying buttress structural behavior is defined by flyer form. In what follows, the criteria for structural efficacy of section 2 will be examined in reference to the geometric parameters of section 3. The results presented here, based on a larger data set, corroborate those published in Nikolinakou et al. (2005).

Figure 6 demonstrates how two geometrical parameters — flyer inclination and the intrados central angle — affect the flyer minimum thrust according to the first criterion. **Figure 6a** shows that the minimum horizontal thrust value decreases as flyer inclination increases: steeper flyers perform better. Flyers which deviate from this trend, for example Saint Martin in Laon, are those with greater thickness, especially at the head. For a given inclination, a thicker flyer head provides the space for a greater rise in the thrust line, resulting in a lower minimum thrust. From this point of view, the thicker the flyer, the better; a solid wall would allow the lowest minimum thrust of all. But a solid wall, of course, does not fly.

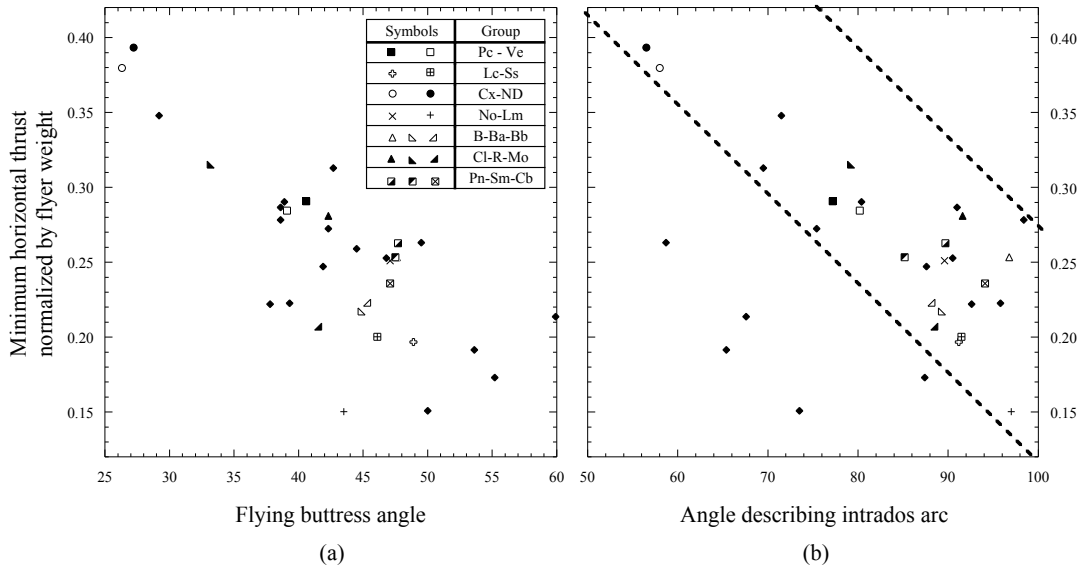


Figure 6. Effect on the minimum horizontal thrust value of a) flyer inclination and b) intrados arc angle. Minimum thrust is normalized by weight. Note that actual minimum thrusts range from 3kN to 45kN (or 120kN, if Notre-Dame in Paris is included)

Figure 6b shows that the minimum thrust decreases as the intrados arc segment increases: flyers with larger arcs perform better. It is interesting to note that the flyers which deviate from the general trend have intrados arc centers which are located well inside the clerestory wall (see the isolated points in the upper right corner of **figure 5**). **Figures 7b** and **8b** show furthermore that the thrust line in flyers whose arc centers are inside the clerestory wall is oriented more vertically at the culée and more horizontally at the head respectively. Such a configuration enhances the flyer's stability according to the second and third criteria in section 2.

Finally, **figures 7a** and **8a** examine the effect of the flyer inclination on the orientation of the thrust line. Steeper flyers, in addition to accommodating lower horizontal thrust values, direct thrusts more vertically into the culée, and thus counteract the general tendency of the flying buttress to induce shear stresses and to overturn the buttress on which the culée sits (**figure 7a**). No clear trend can be observed for the head region (**figure 8a**), however.

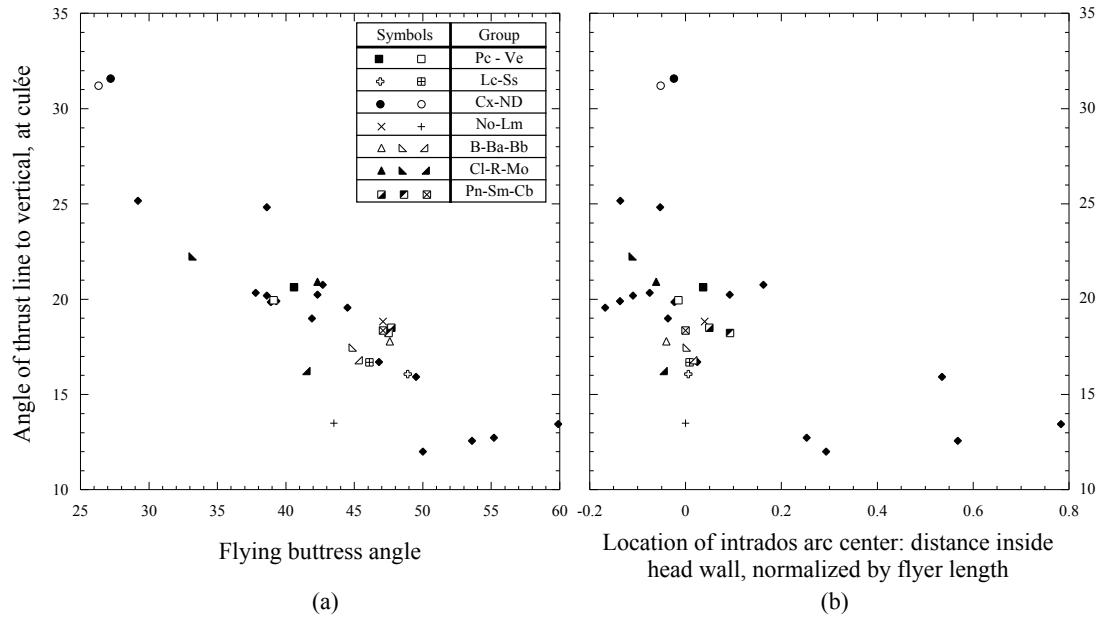


Figure 7. Effect of a) the flyer inclination and b) the location of the intrados arc center on the orientation of the line of thrust at the culée.

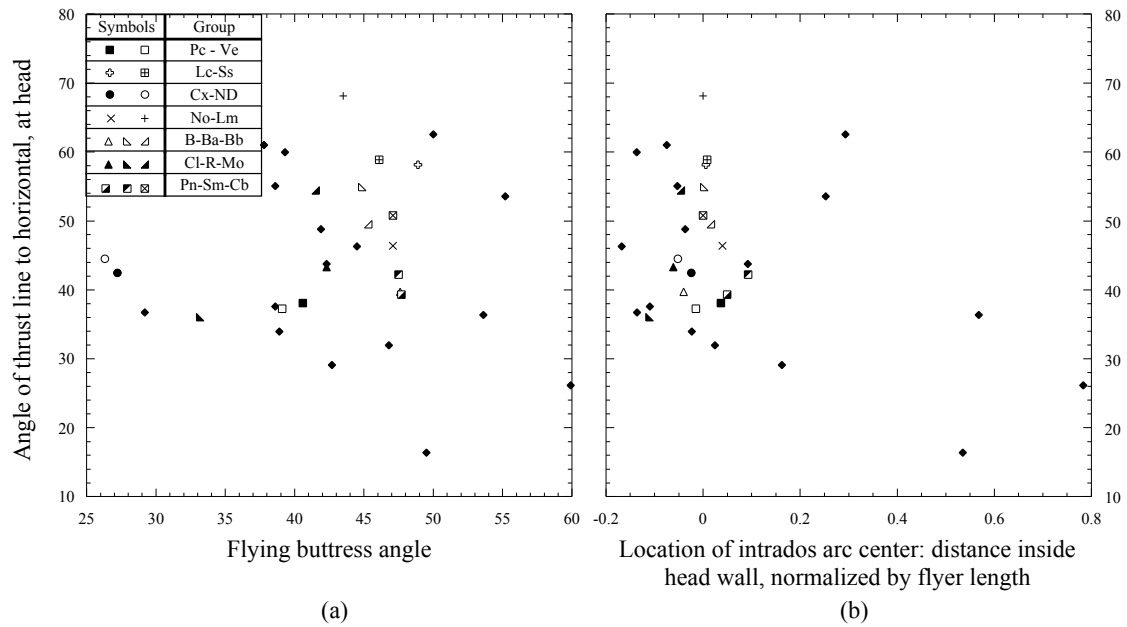


Figure 8. Effect of a) the flyer inclination and b) the location of the intrados arc center on the orientation of the line of thrust at the head.

It should now be clear that certain geometric criteria are linked to structural behavior: head thickness, flyer angle and intrados curvature independently affect the structural performance of the flyer. More striking is that certain formal characteristics affect different aspects of structural performance in a similar way; figure 9, for example, illustrates that increasing the

inclination of a flyer produces a clear combined trend for both the value and the orientation of the line of thrust.

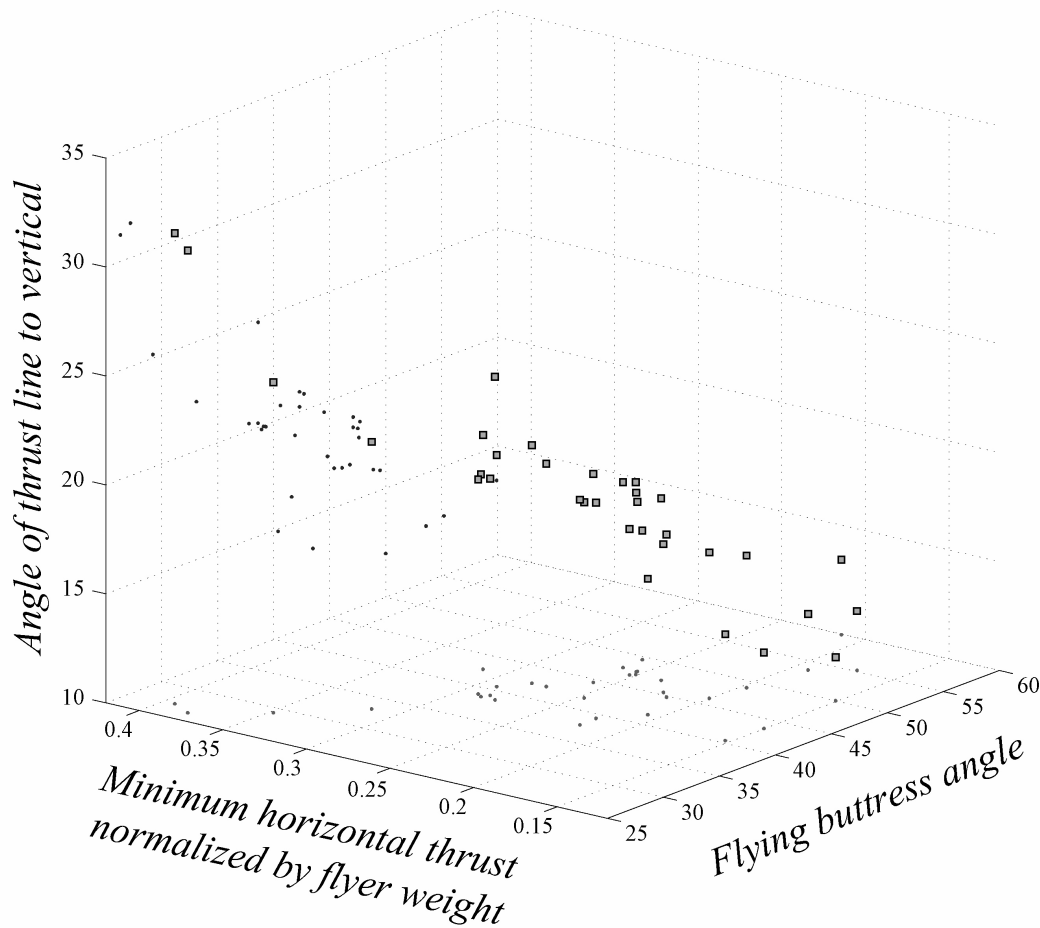


Figure 9. Effect of flyer inclination on both the value of minimum thrust and the orientation of the line of thrust at the culée vicinity. Smaller points are planar projections of the data points in 3D space.

Certain combinations of formal characteristics are more efficient than others, according to the criteria established in section 2. Some of the flyers studied possess such combinations, while others clearly do not. Is this happenstance? Flying buttresses added in the seventeenth or eighteenth centuries to medieval buildings that were originally without — at Saint-Pierre in Bar-sur-Aube, for example — are sometimes mistaken for medieval flyers. They are simple, the simplest self-supporting spanning technology available: an arch with a flat, sloped extrados. Are the trends observed here, then, just minor variations on an extremely simple and obvious concept, the arch-prop? Or do they reveal a conscious manipulation of structure at the hands of the medieval builder? And further: is it possible to know whether the choice of one flyer form over another made with intuition honed by common knowledge of previous structural investigations? Can progress in structural thinking be discerned? Or are these trends rather the fruit of multiple independent experiments?

5. THE CORRELATION OF STRUCTURAL OBSERVATIONS WITH HISTORICAL REALITY

To address these questions, it is necessary to test each observed trend against the historical constraints of twelfth-century France. The careful study of various comparisons of flyer form against structural criteria reveals seemingly valid connections, but also the lack thereof where expected.

a. Strong correlations

The simplest case is that of structural resemblance between two flying buttresses of similar form and dimensions: the flyers in the chevets of the churches of Pontigny and Vézelay (**figure 10, first row**; see also **figures 5-8**), for example. This formal similarity is interesting given that they are both relatively large Burgundian abbey churches with flying buttresses which were built, in the case of Vézelay, as part of the construction of the chevet at the end of the twelfth century, and for Pontigny, as later additions in the thirteenth.

When the flying buttresses of the cathedral of Laon that existed prior to the major nineteenth-century restoration of the building (**figure 10, second row**) are compared with those of the south transept of cathedral of Soissons (also pre-nineteenth century), there is also near-identity in structural behavior, but in this case, despite a difference in actual dimensions. It is because the form is essentially the same (in itself not surprising given that both sets of flyers probably date from the last quarter of the twelfth century and are installed on buildings that lie only 30 km apart): it is as if the builders were each working with a structural “recipe” that was simply scaled to meet the required building proportions.

A further structural correlation exists between the nave flyers at Notre-Dame in Paris as they existed before being replaced in the mid-nineteenth century (**figure 10, third row**) and those of the collegiate church of Saint-Martin in Champeaux — a correlation which is perhaps less obvious given the great difference in scale between the two buildings. It is impossible to say for certain whether these flyers at Notre-Dame were in fact those constructed in the third quarter of the twelfth century. Yet this possibility is strengthened by their structural resemblance to the nave flyers at Saint-Martin, a church which belonged to the diocese of Paris in the twelfth century, with considerable stylistic connections to the cathedral, and whose flyers were probably built in the last quarter of the twelfth century — soon after those in the nave of Notre-Dame. The art-historical significance of this connection will be further discussed in a forthcoming paper.

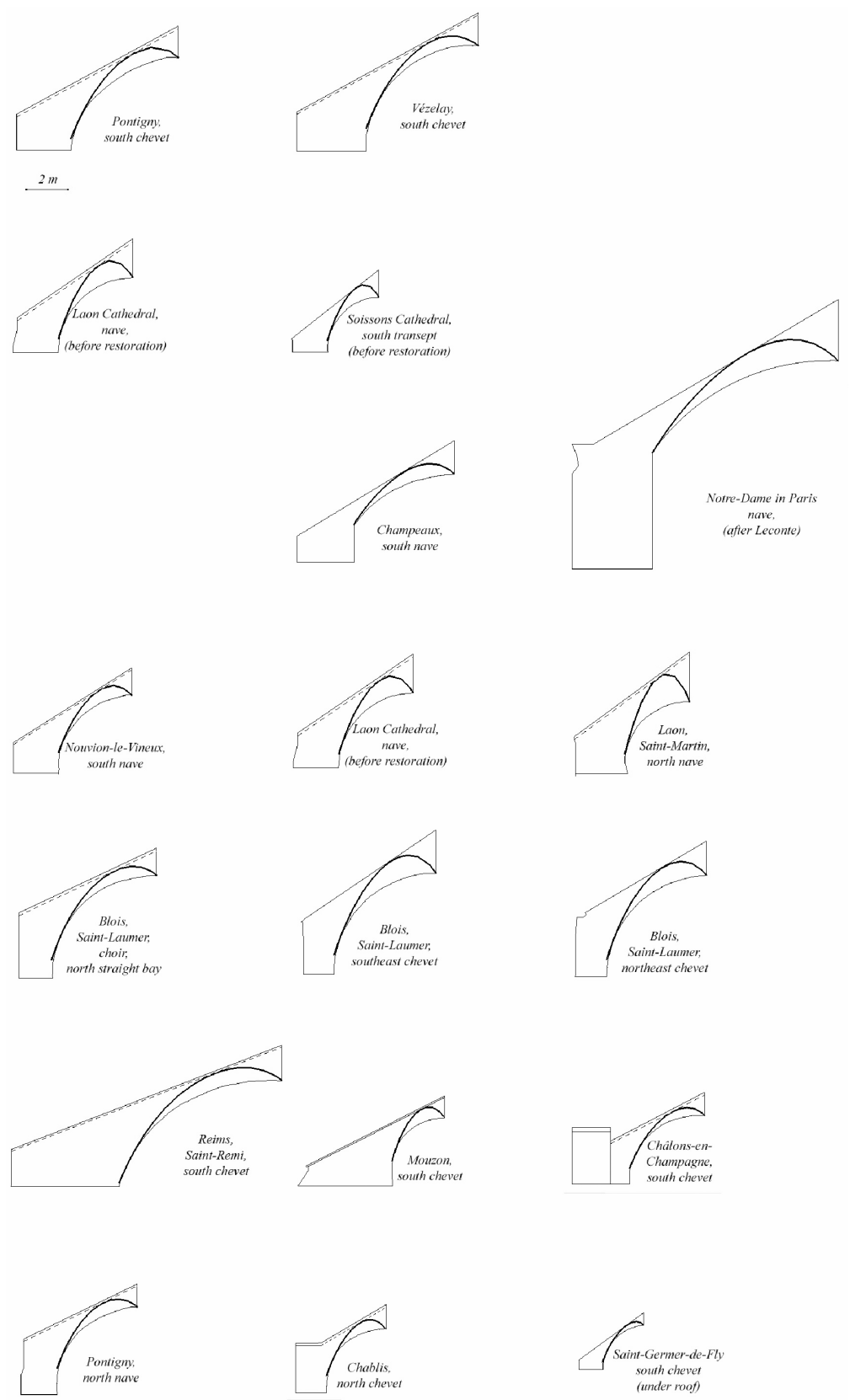


Figure 10. Forms of flying buttresses and their minimum thrust lines. Flyers are drawn to scale

b. Correlations which raise questions

Three sets of flying buttresses were built at nearly the same scale, in nearly the same place and at the same moment at the end of twelfth century: at the cathedral of Laon, the church of Saint-Martin in Laon, and the church of Saint-Martin in Nouvion-le-Vineux, a small town 10 km to the south of Laon. They are interesting to compare because it is reasonable to expect the builders of each to have been familiar with the work of the others. The flyers are roughly similar (**figure 10, fourth row**), but with interesting structural differences. Those at Nouvion present a form which could be considered an improvement on the design of the cathedral — but this difference is subtle enough that it could equally have resulted, for example, simply from a choice to build the centering in a different fashion. The flyers in the nave of Saint-Martin in Laon, however, are quite different structurally. Because of their unusual thickness, they are isolated in nearly every analysis (see **figures 5-10**). They have among the lowest minimum thrust values but are extremely ungainly. Was this design decision by the builder of Saint-Martin in Laon a conscious reaction to a problem perceived in the flyers being built at the nearby cathedral? Could it be attributed to a difference in taste? An indication of practical inexperience and consequent fear-based overdesign? Curiously, the medieval flying buttresses on the cathedral were replaced with far more massive versions in the nineteenth century; the restoration architect considered the original flyers far too frail for the job.

A similar microcosm of flying buttress construction exists in a single building: the church of Saint-Laumer in Blois. The flyers which support the chord of the semidome apse (**figure 10, fifth row**), possibly installed first, are different than those built to support the chevet, which were added while the building was under construction (but not intended from the beginning of construction), and those which abut the first choir bay west of the chord. When the structural data is examined, there *seems* to be an improvement in the design, which corroborates the putative flying buttress constructional sequence — but it must be acknowledged that there is considerable latitude of interpretation. Any such conclusion can be made only with caution.

The churches of Notre-Dame-en-Vaux in Châlons-en-Champagne and Saint-Remi in Reims were constructed virtually simultaneously; the dust has not yet settled over the debate as to which came first and inspired the other. There are strong ties between the buildings, which extend also to the structural system (**figure 10, sixth row**). A further building, the abbey church of Mouzon, built at the very end of the twelfth century, which patterned itself after the cathedral of Laon, but which adopted the buttressing system (on a smaller scale) of Saint-Remi and Notre-Dame-en-Vaux, also has strong connections with these two. An examination of the structural data reveals that, despite the considerable architectural interchange among the three buildings, and despite the general *resemblance* among the three, there is not as strong a structural correlation as might be expected. It is clear that the specifics of flying buttress structure were interpreted by each builder as he saw fit. Perhaps it could even be said that the builder of Mouzon copied the buttress design at Saint-Remi without understanding: Saint-Remi is equipped with a huge culée because it has an extremely long (nearly 8 m) and shallow flyer. The builder of Mouzon retained the deep culée, though his flyer is little over 3 m long, and steeper.

c. Non-correlations

The most difficult case to address is that of a group of three buildings which consistently plot together, and, as shown in **figure 10 (last row)**, have very similar forms, but which are nonnegotiable in historical terms: the nave flyer of the abbey church of Pontigny, added at an indeterminate date after the nave construction was finished in the thirteenth century (possibly as late as the eighteenth century), the below-roof arch at Saint-Germer-de-Fly, from the mid-twelfth century, and the church of Chablis, whose flyers are probably emulations of those at the cathedral of Sens, and probably thirteenth-century additions. Perhaps this total non-correlation is best interpreted as a sign that considerable prudence is required when drawing conclusions based on the analysis methods applied here.

6. CONCLUSIONS

In a previous study (Nikolinakou et al. 2005), based on the presumed structural advantages dictated by minimum thrust analysis, the flying buttress that was considered to perform best overall was one among several at the small church of Voulton. This flyer has similarities with the pre-restoration chevet flyers at Saint-Quiriace in Provins and those of the now-destroyed church of Launay near Sens — but the group stops there. The examples in section 5 are clearly also geographically isolated. It is thus impossible, based on the criteria established, to argue for any sort of extra-regional *progress* — even inter-group refinement, at Blois, for example, must be taken with the greatest caution. Perhaps what we must conclude is that the evidence points to a set of regional experiments that explored specific design ideas not necessarily shared among the entire group — but which nonetheless laid the groundwork for the dramatic structural achievements of High Gothic architecture.

The study of the minimum thrust has revealed a great deal about the structural behavior and construction of the flying buttress arch — but it is not the whole picture. To truly understand the structural nature of the flying buttress, and before any far-reaching historical or geopolitical questions can be answered, the context of the entire building, or at very least the culée, must be embraced. Future work, then, must explore the maximum thrust state, and must look carefully at the acuteness of the extrados angle with respect to the culée, the culée thickness, and the buttress, for example, to place the flying buttress in the largest possible context.

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